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The influence of inelastic relaxation time on intrinsic spin Hall effects in a disordered two-dimensional electron gas

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Abstract
The influence of inelastic relaxation time on the intrinsic spin Hall effects in a disordered two-dimensional electron gas with Rashba interaction is studied; we clarify the controversy as regards impurity effects in the system. We reveal that, due to the existence of inelastic scattering, the spin Hall conductivity does not vanish when the impurity concentration diminishes to zero, regardless of nonmagnetic or magnetic disorder. The spin accumulation is evaluated by using the spin Hall conductivity obtained, and an alternative route is suggested for verifying the intrinsic spin Hall effect by measuring the spin accumulation at different ratios.

1. Introduction

Much attention has been paid to the study of intrinsic spin Hall effects (ISHE), which is expected to bring about practical applications in spintronics. In the intrinsic spin Hall effect, the spin current arises from the spin–orbit-dependent band structure. Theoretically, ISHE may exist in p-type semiconductor [1] and two-dimensional electron gas (2DEG) [2]. After Sinova *et al* [2] predicted a universal spin Hall conductivity, $\sigma_{sH} = e/8\pi$, in clean 2DEG, several groups [3–7] indicated that an arbitrarily small impurity concentration would suppress the spin Hall conductivity to zero due to the vertex corrections. Rashba [6] and Dimitrova [7] proved that the spin Hall current was always zero in the nonmagnetically disordered system while Grimaldi *et al* [8] and Krotkov [9] noticed that the spin Hall conductivity is not zero unless it is in the special situation for a large Fermi circle, quadratic band structure and momentum-independent Rashba coefficient. Very recently, Inoue *et al* [10] and Wang *et al* [11] recognized that the spin Hall conductivity is nonzero in the presence of magnetic impurities. However, no one can explain why there exists a discontinuous jump in σ_{sH} between the clean limit and a clean system. Thus there remains a puzzle as to why the clean limit of the spin Hall conductivity does not equal the one in a clean system. Contradicting analytical results, a numerical calculation for a finite 2DEG by Nomura *et al* [12] manifested a robust spin Hall conductivity that falls to zero only when the inverse of the elastic relaxation time (elastic lifetime for brevity) is larger than the spin–orbit splitting of the bands, i.e., $1/\tau > \Delta$. This raises a question: why is there such a controversy as regards the numerical calculation and the analytical consequences? A consistent comprehension of the aforementioned issues becomes obligatory.

It is known that all the states in two-dimensional infinite systems in the presence of disorder are localized [13] at zero temperature. At finite temperature, there exist delocalized states because of the presence of dephasing. The interference occurs only inside the decoherence length so the electronic conductivity depends on the ratio of elastic to inelastic lifetimes. Although the importance of dephasing in the electrical charge transport in 2D systems has been addressed in the weak localization theory [13–19], there has been no discussion on the role of dephasing in the spin Hall effect. We will show in the present paper that the dephasing plays a crucial role in the spin Hall effect, leading to a nonzero conductivity for an arbitrary impurity concentration.

This paper is organized as follows. In the next section, we give a description of the system and make a general formulation by introducing inelastic relaxation time. In section 3, we investigate the influence on spin Hall conductivity of nonmagnetic impurities. In section 4, we consider magnetic impurities and study their effects on the spin Hall effect. In section 5, we plot the curves of the conductivity versus the ratio of elastic to inelastic relaxation times and observe their asymptotic behaviors. We also discuss the corresponding finite temperature behaviors. In section 6, we evaluate the spin accumulation in terms of the spin Hall

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conductivity that we obtained. Our concluding remarks are briefly summarized in the last section.

2. General consideration

The Hamiltonian for a 2DEG with Rashba spin–orbit coupling is given by $H = p^2/2m^* + \alpha(\sigma^x p_y - \sigma^y p_x) + V_{\rm dis}$ with $V_{\rm dis}$ denoting potentials produced by either nonmagnetic impurities or magnetic impurities. The spin current is defined as

$$\begin{aligned} J_y^z(\mathbf{p}) &= \tfrac{1}{4}(v_y\sigma_z + \sigma_z v_y) \\ &= \frac{1}{2}\frac{p_y}{m^*}\sigma_z. \end{aligned} \tag{1}$$

On the basis of Kubo's formalism, the spin Hall conductivity in response to a dc electric field at zero temperature can be expressed as

$$\sigma_{\rm sH} = -\frac{e}{2\pi}{\rm Tr}\left[J_y^z G^{\rm r}(\mu) j_x G^{\rm a}(\mu)\right], \tag{2}$$

where $j_x = \frac{\rm d}{{\rm d}p_x}(\frac{p^2}{2m^*} + \alpha(\sigma^x p_y - \sigma^y p_x))$ is the electrical current; $G^{\rm r}$ and $G^{\rm a}$ denote the retarded and advanced Green's functions, respectively. The trace is taken over momentum and spin indices.

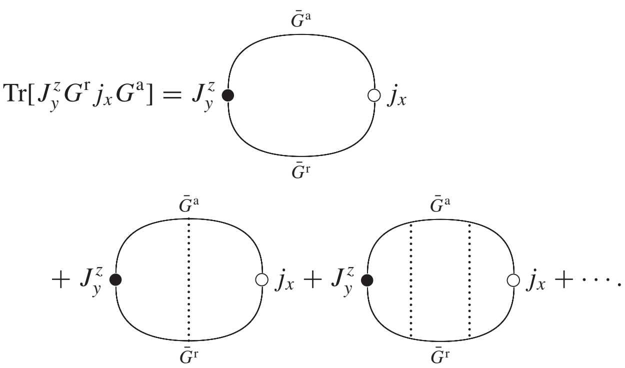
The unperturbed Hamiltonian $H_0 = p^2/2m^* + \alpha(\sigma^x p_y - \sigma^y p_x)$ can be diagonalized as $\varepsilon_\pm(p) = p^2/2m^* \mp p\alpha$ by using the unitary matrix

$$U(\mathbf{p}) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ {\rm i}{\rm e}^{{\rm i}\varphi_{\mathbf{p}}} & -{\rm i}{\rm e}^{{\rm i}\varphi_{\mathbf{p}}} \end{pmatrix},$$

where $\varphi_{\mathbf{p}}$ refers to the azimuthal angle of the momentum $\mathbf{p}$ with respect to the x axis. Then the free particle Green's function in chiral bases can be expressed as

$$G^{\rm r}_{0({\rm ch})}(\mathbf{p},\mu) = \begin{pmatrix} \dfrac{1}{\mu - \varepsilon_+ + {\rm i}\eta} & 0 \\ 0 & \dfrac{1}{\mu - \varepsilon_- + {\rm i}\eta} \end{pmatrix}. \tag{3}$$

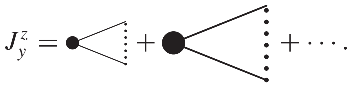
We employ the diagrammatic technique [11] to calculate the average spin Hall conductivity over the distribution of impurities. The trace in equation (2) is expanded as a sum of diagrams:



The spin Hall conductivity consists of two parts $\sigma^0_{\rm sH}$ and $\sigma^L_{\rm sH}$, namely,

$$\begin{aligned} \sigma^0_{\rm sH} &= -\frac{e}{2\pi}{\rm Tr}\left[J_y^z \bar{G}^{\rm r}(\mu) j_x \bar{G}^{\rm a}(\mu)\right], \\ \sigma^L_{\rm sH} &= -\frac{e}{2\pi}{\rm Tr}\left[\tilde{J}_y^z \bar{G}^{\rm r}(\mu) j_x \bar{G}^{\rm a}(\mu)\right], \end{aligned} \tag{4}$$

where the former represents the contribution of the one-loop diagram while the latter arises from the vertex corrections. Here $\tilde{J}_y^z$ refers to the corrected-spin-current vertex which obeys a self-consistent equation [11] illustrated by the following diagram:



The self-consistent Born equation is

$$\begin{aligned} \bar{G}^{\rm r}_{({\rm ch})}(\mathbf{p}) = G^{\rm r}_{0({\rm ch})}(\mathbf{p}) + \sum_{\mathbf{q}} \frac{Nu^2}{V^2} G^{\rm r}_{0({\rm ch})}(\mathbf{p})U^\dagger(\mathbf{p})U(\mathbf{q}) \\ \times \bar{G}^{\rm r}_{({\rm ch})}(\mathbf{q})U^\dagger(\mathbf{q})U(\mathbf{p})\bar{G}^{\rm r}_{({\rm ch})}(\mathbf{p}) \end{aligned}$$

in the presence of nonmagnetic impurities. The Green's function is supposed to be

$$\bar{G}^{\rm r}_{({\rm ch})}(\mathbf{p},\mu) = \begin{pmatrix} \dfrac{1}{\mu - \varepsilon_+ + \frac{\rm i}{2\tau}} & 0 \\ 0 & \dfrac{1}{\mu - \varepsilon_- + \frac{\rm i}{2\tau}} \end{pmatrix}. \tag{5}$$

Then one gets the momentum relaxation time τ, which is related to the impurity concentration n_i and scattering strength u, namely $1/\tau = n_i u^2 m^*$ regardless of the nonmagnetic or magnetic impurities.

To uncover the puzzle and disentwine the controversy in the impurity effects on the spin Hall conductivity, we need to look through the features of an infinite two-dimensional quantum system. An infinite system with infinitesimal impurity concentration contains an infinite number of impurities. Diluting the impurity concentration means increasing the distance between impurities. The infinite quantum systems with different impurity concentrations can be mapped into each other by redefining the length scale and Fermi wavelength, whereas they cannot be directly mapped into a clean system. This means that the clean limit of an infinite quantum system is not always a clean system for the two-dimensional electron gas when the effect of dephasing is ignored. The dephasing can be characterized by inelastic relaxation time.

We employ an imaginary self-energy to represent the inelastic scattering, which was introduced early on in the weak localization theory [16, 19]. Thus the Green's functions $\bar{G}^{\rm r}(\mu)$ and $\bar{G}^{\rm a}(\mu)$ are substituted by frequency-dependent functions $\bar{G}^{\rm r}(\mu + \omega/2)$ and $\bar{G}^{\rm a}(\mu - \omega/2)$ where ω is replaced by ${\rm i}/\tau_i$ with τ_i being the inelastic relaxation time (inelastic lifetime for brevity). As a result, the averaged Green's function in chiral bases turns out to be

$$\bar{G}^{\rm r}_{({\rm ch})} = \begin{pmatrix} \dfrac{1}{\mu - \varepsilon_+ + \dfrac{\rm i}{2\tau} + \dfrac{\rm i}{2\tau_i}} & 0 \\ 0 & \dfrac{1}{\mu - \varepsilon_- + \dfrac{\rm i}{2\tau} + \dfrac{\rm i}{2\tau_i}} \end{pmatrix}. \tag{6}$$

In the limit case $\tau_i \to \infty$, the above Green's function gives rise to equation (5).

3. 2DEG with nonmagnetic impurities

We consider a system in the presence of nonmagnetic impurities. The interaction between electrons and impurities is expressed as

$$V_{\rm dis} = \sum_{i=1}^{N} \int {\rm d}\mathbf{r}^2 u\delta(\mathbf{r}-\mathbf{R}_i)\hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r}). \tag{7}$$

The calculation of the momentum integral of the product of Green's functions is carried out analytically. For simplicity, we adopt the limit of large Fermi energy $\mu \to \infty$ first. The $\sigma^0_{\rm sH}$ that we obtained remains the so called universal value $e/8\pi$.

Furthermore, we calculate the vertex correction. The self-consistent equation for $\tilde{J}^z_y$ reads

$$\tilde{J}^z_y = \frac{n_i u^2}{V}\sum_{\mathbf{p}} \bar{G}^{\rm a}(\mathbf{p},0)(J^z_y(\mathbf{p}) + \tilde{J}^z_y)\bar{G}^{\rm r}(\mathbf{p},0). \tag{8}$$

Solving the above equation with substitution of $n_i u^2$ by $1/(\tau m^*)$, we get the corresponding matrix element, saying

$$(\tilde{J}^z_y)_{\uparrow\downarrow} = \frac{-{\rm i}}{4\alpha m^*(2\tau-\tau')}, \tag{9}$$

where $1/\tau' = 1/\tau + 1/\tau_i$. Then we obtain the vertex correction

$$\sigma^L_{\rm sH} = \frac{-{\rm i}e(\tilde{J}^z_y)_{\uparrow\downarrow} m^*\alpha\tau'}{2\pi} = \frac{-e}{8\pi(2\frac{\tau}{\tau'}-1)}. \tag{10}$$

As a reasonable asymptotic behavior, the corrected-spin-current vertex tends to zero when $\tau \to \infty$ which is the realistic clean limit. In our formulation, the momentum integral of the product of Green's functions $\sum_{\mathbf{p}} \bar{G}^{\rm r}(\mu,\mathbf{p}) j_x(\mathbf{p})\bar{G}^{\rm a}(\mu,\mathbf{p})$ is convergent when $\tau \to \infty$ for a finite τ_i such that the vertex correction goes to zero in the clean limit. This is due to the divergence of the momentum integral when $1/\tau_i = 0$ in the current literature; the vertex correction is not zero in the clean limit. The vertex correction arises from the interference of the scattering waves from different impurities. When the elastic lifetime is much larger than the inelastic lifetime or the distance between impurities is much larger than the decoherence length, the interference effect will disappear. So it is natural that the vertex correction goes to zero in the clean limit, removing the discontinuity in the conductivity variation.

The total spin Hall conductivity is then given by

$$\sigma_{\rm sH} = \frac{e}{8\pi}\frac{1}{1+\frac{\tau_i}{2\tau}}, \tag{11}$$

which fulfills $\lim_{\tau\to\infty} \sigma_{\rm sH} = e/8\pi$ for a finite inelastic lifetime. The clean limit of the spin Hall conductivity is precisely the conductivity in a clean 2DEG due to the disappearance of vertex corrections. Here we do not consider the finite size effect since the system's size is assumed to be much larger than the mean free path or the decoherence length.

In the limit $\tau_i \to \infty$, we have $\sigma_{\rm sH} = 0$ coinciding with the result for a system without inelastic scattering. For large τ/τ_i, the spin transport belongs to the semiclassical regime and the conductivity goes to $e/8\pi$. When τ/τ_i is small, the transport falls into the quantum regime and the conductivity tends to zero. Our result demonstrates the difference between the semiclassical and quantum transport regimes in the spin Hall effect. It was proved that the spin Hall conductivity vanishes when there exist nonmagnetic impurities in a homogeneous system [6, 7]. It was also indicated [6] that inhomogeneities facilitate spin currents. However, the conclusions are not valid after taking account of inelastic lifetime because their Schrödinger equation is not appropriate for describing a system with inelastic scattering. Our consequence is that the spin Hall conductivity is not zero in a realistic system.

In calculating the momentum integrals of Green's functions in the above, we assumed an infinite Fermi energy for simplicity. For a finite Fermi energy, we can also evaluate the spin Hall conductivity in the semiclassical approximation [7, 11]. We get formally a similar $\sigma^0_{\rm sH}$ to [7] with the only difference that τ becomes τ',

$$\sigma^0_{\rm sH} = \frac{e}{8\pi}\left(1 - \frac{1}{1+(\Delta\tau')^2}\right). \tag{12}$$

Now the matrix element of the corrected-spin-current vertex is found to be

$$(\tilde{J}^z_y)_{\uparrow\downarrow} = \frac{-{\rm i}v_{\rm F}\Delta\tau'^2}{4\tau(1+(\Delta\tau')^2) - 2\tau'(1+(\Delta\tau')^2) - 2\tau'}, \tag{13}$$

where Δ is the spin–orbit splitting at the Fermi surface, and $v_{\rm F}$ the Fermi velocity. The vertex correction is obtained as

$$\sigma^L_{\rm sH} = \frac{e}{8\pi}\frac{(\Delta\tau')^2}{1+(\Delta\tau')^2} \times \frac{-\Delta^2\tau'^3}{2\tau(1+(\Delta\tau')^2) - \tau'(1+(\Delta\tau')^2) - \tau'}. \tag{14}$$

Consequently, the total spin Hall conductivity is given by

$$\sigma_{\rm sH} = \frac{e}{8\pi}\frac{(\tau_i\Delta)^2}{(1+\tau_i/\tau)^2 + (\tau_i\Delta)^2(\tau_i/2\tau+1)}. \tag{15}$$

The vertex correction still goes to zero in the clean limit, leading to continuous change of the conductivity with respect to the impurity concentration. For a finite Fermi energy, the spin Hall conductivity depends not only on the ratio of elastic to inelastic lifetimes but also on the spin–orbit splitting at the Fermi surface. The clean limit of equation (15) is $\frac{e}{8\pi}\frac{(\tau_i\Delta)^2}{1+(\tau_i\Delta)^2}$, depending on the inelastic lifetime τ_i. If the inelastic lifetime becomes infinitely long, equation (15) diminishes to zero, recovering the result that we are familiar with.

Let us compare with the numerical result [12]. Nomura *et al* set a finite η^{-1} to guarantee the convergence in their numerical calculation and found that the spin Hall conductivity increases as the impurity concentration decreases and $\eta\tau$ increases. In their paper, η^{-1} is called the electric field turn-on time; in fact, it should be the inelastic lifetime τ_i. Since $\eta\tau$ corresponds to the ratio τ/τ_i, their result supports our present conclusion.

4. 2DEG with magnetic impurities

Very recently, the spin Hall conductivities in the presence of magnetic impurities were calculated in [10, 11] manifesting that the spin transport properties of magnetically and nonmagnetically disordered systems are different. It becomes obligatory to confirm whether this difference remains when the inelastic scattering is not ignored. We adopt the scattering potentials of magnetic impurities as

$$V_{\mathrm{dis}} = \sum_{i=1}^{N} \int \mathrm{d}\mathbf{r}^2\, u\delta(\mathbf{r}-\mathbf{R}_i) \times \hat{\psi}^{\dagger}(\mathbf{r}) \begin{pmatrix} \cos\theta_i & \sin\theta_i \mathrm{e}^{-\mathrm{i}\phi_i} \\ \sin\theta_i \mathrm{e}^{\mathrm{i}\phi_i} & -\cos\theta_i \end{pmatrix} \hat{\psi}(\mathbf{r}). \quad (16)$$

We firstly consider the infinite Fermi energy limit, obtaining $\sigma_{\mathrm{sH}}^{M0} = e/8\pi$. The self-consistent equation for the corrected vertex is expressed as

$$\tilde{J}_y^{Mz} = \frac{n_i u^2}{V} \sum_{\mathbf{p}} \int \mathrm{d}\theta\, \mathrm{d}\phi \frac{1}{4\pi} \sin\theta \times \begin{pmatrix} \cos\theta & \sin\theta \mathrm{e}^{-\mathrm{i}\phi} \\ \sin\theta \mathrm{e}^{\mathrm{i}\phi} & -\cos\theta \end{pmatrix} \times \bar{G}^{\mathrm{a}}(\mathbf{p},0)(J_y^z(\mathbf{p}) + \tilde{J}_y^{Mz})\bar{G}^{\mathrm{r}}(\mathbf{p},0) \begin{pmatrix} \cos\theta & \sin\theta \mathrm{e}^{-\mathrm{i}\phi} \\ \sin\theta \mathrm{e}^{\mathrm{i}\phi} & -\cos\theta \end{pmatrix}. \quad (17)$$

The matrix element of the corrected vertex is found to be

$$(\tilde{J}_y^{Mz})_{\uparrow\downarrow} = \frac{\mathrm{i}}{4\alpha m^*(6\tau+\tau')}. \quad (18)$$

And the vertex correction is obtained as

$$\sigma_{\mathrm{sH}}^{ML} = \frac{e}{8\pi} \frac{1}{6\frac{\tau}{\tau'}+1}. \quad (19)$$

Thus the spin Hall conductivity is given by

$$\sigma_{\mathrm{sH}}^{M} = \frac{e}{8\pi}\left(1 + \frac{1}{6\frac{\tau}{\tau_i}+7}\right). \quad (20)$$

Obviously, the clean limit, $\tau \to \infty$, of the spin Hall conductivity in magnetically disordered systems is also the universal value $e/8\pi$ [2]. In the limit of infinite inelastic lifetime, σ_{sH}^{M} goes to $e/7\pi$ consistently with the recent results [10, 11] for magnetic impurities.

We also consider the case for a finite Fermi energy. The $\sigma_{\mathrm{sH}}^{M0}$ here retains the same result as equation (12) for nonmagnetic impurities, while the matrix element of the corrected vertex reads

$$(\tilde{J}_y^{Mz})_{\uparrow\downarrow} = \frac{\mathrm{i}v_{\mathrm{F}}\Delta\tau'^2}{12\tau(1+(\Delta\tau')^2) + 2\tau'(1+(\Delta\tau')^2) + 2\tau'} \quad (21)$$

and the vertex correction is given by

$$\sigma_{\mathrm{sH}}^{ML} = \frac{e}{8\pi}\frac{(\Delta\tau')^2}{1+(\Delta\tau')^2} \times \frac{\Delta^2\tau'^3}{6\tau(1+(\Delta\tau')^2) + \tau'(1+(\Delta\tau')^2) + \tau'}. \quad (22)$$

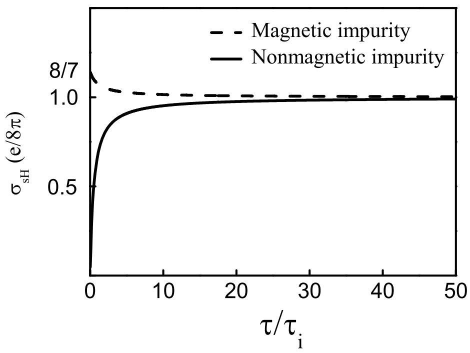


Figure 1. Spin Hall conductivity versus the ratio of elastic to inelastic lifetimes is plotted in both cases for nonmagnetic and magnetic impurities.

The total spin Hall conductivity in the presence of magnetic impurities is then expressed as

$$\sigma_{\mathrm{sH}}^{M} = \frac{e}{8\pi}\frac{(\tau_i\Delta)^2}{(1+\tau_i/\tau)^2 + (\tau_i\Delta)^2\frac{7+6\tau/\tau_i}{8+6\tau/\tau_i}}. \quad (23)$$

When $\tau/\tau_i = 0$, equation (23) becomes $\frac{e}{8\pi}\frac{8(\Delta\tau)^2}{8+7(\Delta\tau)^2}$, recovering the result of our recent paper [11]. When $\tau/\tau_i \to \infty$, equation (23) reduces to $\frac{e}{8\pi}\frac{(\tau_i\Delta)^2}{1+(\tau_i\Delta)^2}$ which is precisely the conductivity in the clean limit of nonmagnetic impurities for finite Fermi energy.

5. Discussion of the impurity effects

The above studies exhibited that the spin Hall conductivities, in the presence of either nonmagnetic or magnetic impurities, depend on the ratio of elastic to inelastic lifetimes. We plot the curves of the conductivity versus τ/τ_i in the presence of nonmagnetic and magnetic impurities in figure 1. As the impurity concentration decreases (i.e., τ increases), σ_{sH}^{M} decreases while σ_{sH} increases monotonically. The magnetic impurities enhance the ISHE, while the nonmagnetic impurities suppress it. In the clean limit, both magnitudes approach $e/8\pi$. In the dirty limit or infinite inelastic lifetime limit, σ_{sH}^{M} goes to $e/7\pi$ but σ_{sH} goes to zero.

It is worthwhile to consider the finite temperature case. We have already shown that the spin Hall conductivity depends on the ratio of elastic to inelastic lifetimes merely in the approximation of infinitely wide band and infinitely large Fermi energy. These two characteristic times are important to both the charge and spin transport properties for 2D systems. Our results provide a clue to how to compare those characteristic times by making use of the charge and spin transport experiments. At low temperature, the elastic lifetime τ is determined by the impurity concentration and is a constant independent of the temperature, while the inelastic lifetime τ_i relates to the electron–electron interaction and decreases when the temperature increases. In general, $\tau_i \propto T^{-p}$ whose exponent p depends on the scattering mechanism (e.g., $p = 2$ for electron–electron scattering); then the spin Hall conductivity is proportional to $1/(1+KT^{-p})$, increasing as the temperature rises. Since the spin Hall conductivity is sensitive to the temperature, its temperature dependence is expected to distinguish the ISHE and ESHE.

6. Spin accumulation

The spin accumulation brought about by spin Hall effect has been observed [20–25] in experiments. We therefore evaluate the spin accumulation generated by the intrinsic spin Hall current on the basis of the results that we obtained in the above. We consider a bar of width W in the y-direction with an applied electric field along the x-direction, for which a spin current along the y-direction will arise. If the width is much larger than the mean free path and the decoherence length $W \gg l,\ l_\varphi$, the system is in the diffusive transport regime. The spin accumulation can be studied using the diffusion equation

$$D\frac{\mathrm{d}^2 S_z}{\mathrm{d}y^2} = \frac{S_z}{\tau_\mathrm{s}}, \tag{24}$$

where $D = v_\mathrm{F}^2\tau/2$ is the diffusive coefficient, τ_s the spin relaxation time (spin lifetime for brevity), and $S_z = 1/2(n_\uparrow - n_\downarrow)$. The spins accumulate at the edges of the bar until the diffusive spin current in opposite direction balances the spin Hall current. Then a steady spin distribution is established. For simplicity, we do not take the influence of the spin accumulation on the spin Hall current into account. The boundary condition is then given by

$$J_y^z = D\frac{\mathrm{d}S_z}{\mathrm{d}y}\bigg|_{y=\pm W/2}. \tag{25}$$

The solution of the above diffusion equation is

$$S_z(y) = \frac{J_y^z\sqrt{\tau_\mathrm{s}/2D}}{\cosh\frac{W}{\sqrt{2D\tau_\mathrm{s}}}}\sinh\frac{y}{\sqrt{D\tau_\mathrm{s}/2}}. \tag{26}$$

Equation (26) is a good approximation for the spin distribution in the region away from the edge. Approaching the edge, however, the spin accumulation will affect the spin current significantly, leading to a dramatic loss of spin current. According to equation (26), the spin density at the edge of the sample is

$$S_z = \frac{eE}{8\pi v_\mathrm{F}}\frac{2\tau/\tau_i}{2\tau/\tau_i + 1}\sqrt{\frac{\tau_\mathrm{s}}{\tau}}\tanh\frac{W}{v_\mathrm{F}\sqrt{\tau\tau_\mathrm{s}}} \tag{27}$$

in the presence of nonmagnetic impurities, and

$$S_z = \frac{eE}{8\pi v_\mathrm{F}}\frac{6\tau/\tau_i + 8}{6\tau/\tau_i + 7}\sqrt{\frac{\tau_\mathrm{s}}{\tau}}\tanh\frac{W}{v_\mathrm{F}\sqrt{\tau\tau_\mathrm{s}}} \tag{28}$$

in the presence of magnetic impurities. Here E stands for the electric field in the x-direction. The spin accumulation depends on the three characteristic times: the elastic lifetime, the inelastic lifetime and the spin lifetime. Because the different characteristic times can be manipulated separately, our result proposes an alternative route to verifying the ISHE by measuring the spin Hall accumulation under different conditions.

The spin accumulation due to the spin Hall effect was analyzed theoretically [25–28]. In the 2DEG, the spin accumulation was observed in [24], where the authors regarded it as arising from ESHE because the relation $\Delta < 1/\tau$ is satisfied in their experiments. In this situation, they believe that the ISHE does not exist according to Nomura's calculation [12]. But our result shows that the spin Hall conductivity is not zero; instead, it depends on the characteristic times of the system, especially on the ratio of elastic to inelastic lifetimes. Thus a careful discussion is necessary for one to judge whether the observed spin accumulation arises from the ESHE or from the ISHE. An experiment at different temperature is expected to be helpful.

7. Concluding remarks

We indicated that the clean limit of an infinite quantum system is not a clean system if the effect of dephasing is ignored. It was hence natural for us to have obtained a discontinuity (zero with arbitrarily small impurity concentration but $e/8\pi$ with no impurities) in the spin Hall conductivity for infinite systems without taking account of dephasing which is characterized by inelastic relaxation time. The disagreements between numerical result and the other analytical results become inevitable because the former [12] dealt with a finite system and the clean limit of a finite system is a clean system.

We exposed the influence of the inelastic relaxation time on the ISHE for 2D electrons in the presence of magnetic and nonmagnetic impurities. We found that the inelastic scattering plays an important role in the spin Hall effect, leading to a nonzero spin Hall conductivity for arbitrary impurity concentrations. In the dirty limit, the spin Hall conductivity goes to zero and $e/7\pi$ for nonmagnetic or magnetic impurities, respectively. It tends to $e/8\pi$ in the clean limit regardless of the magnetic or nonmagnetic disorder of the systems. We revealed the importance of characteristic times, such as the elastic, inelastic and spin lifetimes for the ISHE. The spin Hall conductivity is shown to depend on the ratio of elastic to inelastic lifetime and varies when temperature changes, which provides a method for distinguishing the ISHE and ESHE by measuring the spin current at different temperatures. On the basis of the spin Hall conductivity that we obtained, we evaluated the spin accumulation and presented an alternative route to verifying the ISHE by measuring it under different conditions.

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